Competition among different levels of government: the re-election problem

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Abstract: The aim of this paper is to analyse competition between two levels of government that want to maximise their tax revenues facing the problem of re-election. We assume that citizens have incomplete information about central and local public goods. Then, they are not able to choose a single efficient level of these two goods but they are able to choose a set of combinations. This choice represent the constraint faced by the levels of government. We develop a model starting from this assumption and three extensions to analyse the behaviour of two levels of government in different scenarios. We find that in each situation the equal sharing of the tax revenue is not the best solution for the governments.

JEL classification: D43, D72, H11, H71, H77.
1. Introduction

The aim of this paper is to analyse the behaviour of two different tiers of government that compete to maximize their tax revenues. Governments want to maximize their tax revenues, as firms want to maximize their profits. This is a simple but probably well-grounded assumption often adopted in the literature (Breton and Ursprung, 2002). Then, using the theory of the oligopoly, we develop a model that describes the behaviour of governments that have to sell a particular good, their re-election.

We consider that the local government and the central government provide a unique public good (or two public goods that are perfect substitutes, because it does not change our model) using their tax revenues. In addition, we assume that governments may extract a proportional rent from their tax revenues. The rent share of the tax revenue, \( \alpha \) (with \( 0 < \alpha < 1 \)), is the same for each level of government.

The effect of vertical competition on taxation was developed by Flowers (1988), who improved the model of Brennan and Buchanan (1980) that considered the horizontal competition as a way to reduce the level of taxation with respect to a tax revenue-maximising government (the Leviathan). Flowers showed that vertical competition does not have the same effect. In fact two governments that maximize their tax revenues imposing two taxes on the same tax base increase the total amount of taxes paid by citizens who live in the same jurisdiction as compared to a single government. Flowers did not consider the problem of the re-election for the two governments.

It is assumed that the utility of the government is equal to the share \( \alpha \) of the tax revenue of the current period in office plus a re-election factor \((R)\). The value of the re-election is equal to the expected rent of the next period. Citizens re-elect a government if and only if its tax rate is not higher than a given value \((\tau^*)\).

Using these assumptions we analyse when a government find it profitable to maximise its rent in the current period or to maximise it for an infinite period. In the first case the preferences of citizens are always neglected, in the second they are always respected.

In section 2 a model with one government as a benchmark is considered. In section 3 we introduce two levels of government that compete for the rents and in section 4 a numerical example is provided. In section 5 we introduce progressive taxation and in section 6 we compare proportional with progressive taxation. In section 7 we describe the case where there exist different localities with different median voter’s preferences. In section 8 we consider a non-linear re-election constraint and in section 9 some conclusions will be outlined.
2. One level of government

Starting with a single government providing a single public good, equation (1) shows the rent of the government, assuming that revenue comes from a proportional income tax, the rent is a share ($\alpha$) of the total revenue and $R$ is a given value. As $\alpha$ is constant, if we maximize the tax revenue we are maximizing the rent of government, so we will not use the parameter $\alpha$ in the rest of this paper after equation (1).

$$\alpha W = \alpha \tau Y + \alpha R$$ \hfill (1)

Now we endogenize the re-election value ($R$). $R$ represents the discounted value of the tax revenue ($G$) from period $t = 1$ to $t = \infty$ if a government is re-elected.

$$R = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} \cdot G = \frac{1}{r} \cdot G$$ \hfill (2)

where $r$ is the discount rate of the period of office of the legislature.

As assumed before, citizens re-elect a government if its tax rate is not higher than $\tau^*$. Then $R$ in equation (2) represents the value of the discounted revenue if the re-election constraint holds. Otherwise the value of $R$ is 0. The government chooses to be re-elected if

$$\left(1 + \frac{1}{r}\right) \cdot \tau^* Y^* \geq \tau Y.$$ \hfill (3)

where $\tau$ represents the tax rate imposed by a monopolistic government (Leviathan) and $Y$ is the income associated to the Leviathan tax rate. $Y^*$ represents the income associated to the re-election constraint $\tau^*$. We assume that $\tau^* < \tau$, and $\tau^* Y^* < \tau Y$ by definition. We multiply $\tau^* Y^*$ by $\left(1 + \frac{1}{r}\right)$ because we consider an infinite period.

Government will opt for re-election if $r$ is low and the value of $\tau^* Y^*$ is not so far from the value of $\tau Y$.

1 The value of $Y$ is function of $\tau$. As $\tau$ increases $Y$ decreases.
As assumed before, $r$ represents the discount rate associated to the period of office of the legislature, (4 or 5 years as a rule). If $t = \infty r \rightarrow \infty$ and the left-hand side of (3) tends to $\tau^* Y^*$, which is lower than $\overline{\tau Y}$ by definition. Then, the government chooses the Leviathan tax rate because it has no incentives to behave in the interest of citizens. In a democracy the term between two elections is never $t = \infty$. Then we suppose that the Leviathan tax rate could be useful to describe a dictatorship or, even better, a benchmark, because a dictator needs a given level of consensus to stay in office (Wintrobe, 1998).
3. Two levels of government

We now assume the presence of two levels of government. The median voter imposes a single constraint to both governments. We assume a low level of information, where the median voter perceives the local public good and the central public good as perfect substitutes. Then, the re-election constraint has the following shape:

\[ \tau_L + \tau_C \leq \tau^*, \]  

(4)

where \( \tau_L \) is the local tax rate, \( \tau_C \) is the central tax rate and \( \tau^* \) is the re-election constraint for both governments. Our median voter considers only the tax rates in his constraint because we assume that taxes are proportional.

We analyse \( R \) as endogenous, and we assume that:

\[ (1 + \frac{1}{r}) \cdot \tau^* Y^* \geq \tau^{LS} Y^{S} + \tau^{FS} Y^{S}. \]  

(5)

where \( \tau^{LS} \) is the tax rate of the leader and \( \tau^{FS} \) is the tax rate of the follower in a Stackelberg model. \( Y^{S} \) is the income of the Stackelberg solution. \((\tau^{LS} + \tau^{FS}) > \tau, Y^{S} < \bar{Y}\) and \((\tau^{LS} + \tau^{FS}) Y^{S} < \bar{Y}\) by definition.

We use a particular version of the Stackelberg model to represent our situation without re-election because one government has an advantage if it is the first player who chooses the tax rate. We assume that the central government is the first mover because it has no coordination costs. Then, the central government is the leader in our model.

If we consider formula (5) as an equation, there is a single allocation that allows the re-election of the two governments: the equivalent of the Stackelberg solution in an infinite horizon. If we consider the situation, plausible in a democracy\(^2\), where the left-hand side is higher than the right-hand side in (5), there is competition between the two levels of government to maximise their own tax revenues.

Then, if we consider an advantage for the central government, we know that it maximizes its tax rate leaving to the local government the lowest tax rate that allows its re-election.

\(^2\) See the end of the paragraph
The next equation shows the share of the total tax rate that the central government leaves to the local government:

\[ \tau_L = \left(1 + \frac{1}{r}\right) \cdot \frac{\tau^* Y^*}{\gamma} = \tau^F S Y^S, \quad (6) \]

and \( \gamma \)

\[ \gamma = \left(1 + \frac{1}{r}\right) \cdot \frac{\tau^* Y^*}{\tau^F S Y^S}. \quad (7) \]

The best allocation for the central government occurs when the local government chooses a tax rate equal to \( \frac{\tau^*}{\gamma} \) and it is re-elected. In this case the central government can maximise its tax revenue choosing a tax rate equal to \( \frac{(\gamma - 1)\tau^*}{\gamma} \) to be re-elected. In fact, the local government is indifferent between the choice of \( \frac{\tau^*}{\gamma} \) with re-election or \( \tau^F S \) without re-election.

We can use game theory to show that the solutions that we previously found are stable. We have a sequential game where the central government offers to the local one to impose a tax rate \( (0 \leq \tau_x \leq \tau^*) \) and the local government can accept or reject the offer. If our local government rejects the offer the game ends and each government earns the Stackelberg tax revenue as shown before. If the local government accepts the offer its pay-off is \( \tau_x Y^* \). The central government’s pay-off is \( (\tau^* - \tau_x) Y^* \). Moreover, if the offer is accepted the game continues with a second stage that is equal to the first using a discount factor \( \delta = \frac{1}{1 + r} \). Our game becomes infinite if the local government always accepts the offer of the central one. We use this framework because we assume that parties have a complete turnover if and only if they lose an election.

In figure 1 we show a stage of our game.
The unique Subgame Perfect Nash Equilibrium \( \left( \frac{1}{1-\delta} (\tau^*-\tau_s)Y^*; \tau_s Y^* \right) \) occurs when:

\[
\frac{1}{1-\delta} \tau_s, Y^* = \tau^{FS} Y^*.
\]  

Therefore, from equation (6) and equation (8):

\[
\tau_s = \frac{\tau^*}{\gamma}.
\]  

The solution showed in equation (9) tells us that the local government is indifferent between “accept forever” and “reject”. If the local government chooses the first strategy (“accept forever”), the central government maximises its pay-off.

We may conclude that:

\[
\tau_{L}^* = \frac{\tau^*}{\gamma},
\]  

and:
\[ \tau_c^* = \frac{\gamma - 1}{\gamma} \tau^*. \] (11)
4. A numerical example

If we use an explicit form of the relation between the income and tax, we can solve the problem presented before using distortionary taxes, where a higher tax rate leads to a lower revenue. We use a simple formula without loss of generality, because we know that a more complex functional form leads only to complex computations:

\[ Y = 100 - 100(\tau_L + \tau_C) \quad (12) \]

furthermore we assume:

\[ \tau_L + \tau_C = \tau^* = 0.3 \quad \text{and} \quad r = 0.5 \]

The local government is the follower in our Stackelberg model. If we assume that the tax collection has a cost equal to 0, we may write the reaction function of the local government:

\[ \tau_L = \frac{1 - \tau_C}{2} \quad (13) \]

Then we maximize the tax revenue of the central government:

\[ \max_{\tau_C} \tau_C Y = \tau_C \left( 100 - 100\tau_C - 100 \frac{1 - \tau_C}{2} \right) \quad (14) \]

and we obtain:

\[ \tau_C = \frac{1}{2} \quad (15) \]

and, using the reaction function:

\[ \tau_L = \frac{1}{4} \quad (16) \]
We compute $Y^S = 25$ and we show that the tax revenue for the local government ($\tau^{FS}Y^S$) is 6.25 and the tax revenue for the central government is ($\tau^{LS}Y^S$) is 12.5.

Then, we have to compare the level of the tax rate that allows the same tax revenue for the local government in a situation where the two governments are re-elected to the Stackelberg solution.

From (12) $Y$ is 70 if $\tau_L + \tau_C = \tau^* = 0.3$. In terms of equation (6) we can rewrite the problem of the leader:

$$
\left(1 + \frac{1}{0.5}\right) \cdot \frac{0.3 \times 70}{\gamma} = 6.25.
$$

From (7):

$$
\gamma = \left(1 + \frac{1}{0.5}\right) \cdot \frac{0.3 \times 70}{6.25} = 10.08
$$

$\tau_L = 0.0298$ and $\tau_C = 0.2702$. The tax revenue of our central government is:

$$
\left(1 + \frac{1}{r}\right) \cdot \frac{(\gamma - 1) \cdot \tau^* Y^*}{\gamma} = 56.75.
$$

that is equal to 56.75. This value is higher than 12.5 (the Stackelberg tax revenue for the leader when it does not care about re-election).

The last question of this paragraph is: what is the lower level of $\tau^*$ that allows the re-election of both governments? We have to consider formula (4) as an equation and compute:

$$
\left(1 + \frac{1}{0.5}\right) \cdot \tau^* (100 - 100\tau^*) = 12.5 + 6.25.
$$

We obtain $\tau^* = 0.067$.

As argued before this tax rate is low and no western democracy has a so low tax rate.
If we consider the presence of a single government, it accepts to be re-elected if the value of $\tau^* \geq 0.092$. Consequently, citizens may impose a stronger constraint and governments will have an incentive in behaving yieldingly as the number of tiers increases.

This is a general result. An aggressive government can behave as a monopolist and earn the maximum tax revenue only when it is alone. As the number of tiers increases, the level of market power decreases and governments accept a lower total amount of the tax rates to behave yieldingly.
5. Progressive taxation

We now assume that all governments have to impose a progressive income tax (as it occurs in all western democracies). We assume a linear (negative) income tax, with constant marginal tax rate. Then:

per capita local income tax revenue: $-\bar{L} + \tau_L Y$ \hspace{1cm} (21)

and

per capita central income tax revenue: $-\bar{C} + \tau_C Y$ \hspace{1cm} (22)

Therefore the total tax paid by a citizen is:

$$-\bar{L}_L + \tau_L Y - \bar{L}_C + \tau_C Y,$$ \hspace{1cm} (23)

where $\bar{L}_L$ and $\bar{L}_C$ represent fixed subsidies. A Constitutional law should fix them because governments can easily change a simple law. If we have positive values of $\bar{L}_L$ e $\bar{L}_C$ the average tax rate is increasing and the income taxes are progressive. If a government wants to maximise its tax revenue, it chooses the lowest level of $\bar{L}_i$ (with $i = L, C$) because the higher is the level of progressiveness, the higher is the distortion of the taxes and the lower is the rent. Therefore a minimum level of $\bar{L}_i$ imposed by Constitutional law must be chosen by both levels of government. Then, we know that both governments choose only their tax rates, as in the case of proportional taxation. Nevertheless, the result is different.

The re-election constraint is:

$$\tau^* Y = -\bar{L}_L + \tau_L Y - \bar{L}_C + \tau_C Y$$ \hspace{1cm} (24)

where $\tau^*$, represents the higher average tax rate accepted by the median voter to re-elect the two governments and $Y$ is the median voter’s income associated to $\tau^*$.

We can divide equation (24) by $Y$ and we obtain:
\[ \tau^* = -\frac{Y_L + Y_C}{Y} + \tau_L + \tau_C, \quad (25) \]

and:

\[ \tau^* + \frac{Y_L + Y_C}{Y} = \tau_L + \tau_C. \quad (26) \]

Our constraint has three new terms as compared to the proportional tax rate. We know that \( Y_L \) and \( Y_C \) represent two constant terms and \( Y \) is a variable. In particular, in localities where the median voter has a high income, the left-hand side and the right-hand side in equation (26) are lower with respect to a locality with a poor median voter. This scenario occurs in a situation of progressive taxation because a median voter with a high income has an average tax rate that is higher than a median voter with a low income. Therefore he wants a lower taxation. As argued by Breton and Ursprung (2002) we know that this effect can be balanced by the willingness to pay taxes of rich to reach higher levels of public goods with respect to poor. While in the model that considers proportional taxation the re-election constraint is exogenous, in our model of progressive taxation the dependency of the re-election constraint on the income level emerges.
6. A comparison between proportional and progressive taxation

Our constraint is equal to \( \tau^* \) with proportional taxation and \( \tau^* + \frac{Y_L + Y_C}{Y} \) with progressive taxation, assuming \( \frac{Y_L + Y_C}{Y} > 0 \).

If the central government can get the lion’s share of the tax rate, with a fixed rate of \( \gamma \), the local tax rate is:

\[
\tau_L^{**} = \frac{\tau^* + \frac{Y_L + Y_C}{Y}}{\gamma}
\]  

(27)

and the central tax rate is:

\[
\tau_C^{**} = \frac{(\gamma - 1) \left( \tau^* + \frac{Y_L + Y_C}{Y} \right)}{\gamma}
\]  

(28)

To compute the local tax and the central one we have to consider the fixed subsidies \( (Y_i) \). Therefore we obtain that the average local tax rate is:

\[
-\frac{Y_L}{Y} + \frac{\tau^* + \frac{Y_L + Y_C}{Y}}{\gamma}
\]  

(29)

and the average central tax rate is:

\[
-\frac{Y_C}{Y} + \frac{(\gamma - 1) \left( \tau^* + \frac{Y_L + Y_C}{Y} \right)}{\gamma}
\]  

(30)

We note that the proportion between the two tax rates is influenced by the amount of the subsidies. Then, it is possible to rule the distribution of the income tax between the two levels of
government. In particular, if we think that the local government is penalized as compared to the central one, the Constitution might provide that the central government must impose progressive taxation when the local one is allowed to choose proportional. This could be a way to redistribute the tax revenue between the two levels of government. Moreover, in this case, the redistributive function would be fulfilled by the central government, as the classical theory of decentralization affirms (Musgrave, 1959) and recent analyses confirm (Giarda, 1995 and 2003).

As we said in the fourth paragraph, progressive taxation can be useful to explain the differences in the choice of the re-election constraint among localities. Also the presence of different preferences among the median voters situated in different localities may explain the point.
7. Different re-election constraints among localities

Now we analyse a context where preferences of citizens are different in different localities. Then the level of $\tau^*$ is not a constant in the whole country. Moreover, we assume that our central government has to choose a unique tax rate for its whole jurisdiction. Then, it has to face different constraints in different localities but a single choice for the level of $\tau_C$.

$\tau_1$, $\tau_2$ and $\tau_3$ respectively represent the re-election constraint of locality 1, locality 2 and locality 3. We assume that $\tau_1 > \tau_2 > \tau_3$ (figure 2). A central government is re-elected if it wins the elections in 2 localities out of 3.

Figure 2

The central government rent-maximization problem is illustrated by resorting two numerical examples. Let us reconsider equation (12) as shown in section three:

$$Y = 100 - 100(\tau_L + \tau_C).$$

We can note that the distortion of these taxes is function of the sum of the two tax rates. For simplicity we assume that the three localities have the same level of income since different incomes do not affect our conclusions. Moreover we assume that the discount rate of the period of office of the legislature ($r$) is equal to 0.5 and three re-election constraints as: $\tau_1 = 0.4$, $\tau_2 = 0.3$ e $\tau_3 = 0.2$.

Starting from equation (6) and equation (21) we have the three values of $\gamma$ associated to the three re-election constraints ($\gamma_1 = 11.52$, $\gamma_2 = 10.08$ and $\gamma_3 = 7.68$). Then, the three local tax rates are $\frac{\tau_1}{\gamma_1}$.
0,0347, \( \frac{\tau_2}{\gamma_2} = 0,0298 \) e \( \frac{\tau_3}{\gamma_3} = 0,0260 \). Now, we have to test if it is better for a central government to be re-elected in 2 localities out of 3 rather than in all 3 localities.

If the central government wants to win in two localities, it has to choose a tax rate equal to \( \tau_2 - \frac{\tau_2}{\gamma_2} \) (in our case 0,2702). In fact, it wins in locality 1, where the local government could accept a central tax rate equal to \( \tau_1 - \frac{\tau_1}{\gamma_1} \) (in our case 0,3653) and in locality 2 (the median locality). Locality 3 chooses not to re-elect the central government if the local one chooses to impose a tax rate higher than \(-0,070 \left( \frac{\gamma_2 - 1}{\gamma_2} \right) \). A negative tax rate leads to a negative tax revenue and, consequently, to a negative rent. Therefore the local government always chooses to maximize its rent for a single period of office of the legislature, loosing the next election:

\[
\max_{\tau_L} \tau_L Y_L = \left[ 100 - 100 \left( \tau_L - \frac{\gamma_2 - 1}{\gamma_2} \right) \right] \tau_L
\]

(31)

In this case the local government’s tax rate is \( 1 - \frac{\gamma_2 - 1}{\gamma_2} \) that is equal to 0,3649 considering equation (12). Its tax revenue is equal to 13,31.

The central government earns a tax revenue equal to \( \frac{\gamma_2 - 1}{\gamma_2} \tau_1 Y_1 \) in locality 1, \( \frac{\gamma_2 - 1}{\gamma_2} \tau_2 Y_1 \) in locality 2 and \( \frac{\gamma_2 - 1}{\gamma_2} \tau_2 \) multiplied by the revenue that is related to the choice of the local governments in locality 3 as previously shown. The tax revenue of our central government is equal to 44,99.

We now analyse the problem of a central government that wants to be re-elected in each locality. In this case, the local government of locality 3 has to behave yieldingly. The imposition of a tax rate equal to \( \frac{\gamma_3 - 1}{\gamma_3} \) is not sufficient to obtain a yielding behaviour. In fact this local government can keep an advantage in behaving aggressively in the current period (in the future it will be voted out of office). In this case it earns a tax revenue equal to 17,05. In contrast, if it
accepts that the central government chooses a tax rate equal to $\frac{\gamma_3 - 1}{\gamma_3} \tau_3$ imposing $\frac{\tau_3}{\gamma_3}$, it earns 6,25. Equation (32) shows the problem faced by the central government if it wants the re-election also in locality 3. It has to convince the local government to behave yieldingly.

$$\left( 1 + \frac{1}{r} \right) (r_3 - r_c^*) \left( A - A \cdot r_3 \right) = \frac{1 - r_c^*}{2} \left( A - A \left( \frac{1 - r_c^*}{2} + r_c^* \right) \right).$$

(32)

The left-hand side of (32) represents the utility for the local government of locality 3 if it behaves yieldingly. The right-hand side represents its utility if it behaves aggressively. Starting from equation (32) we can derive the value of $r_c^*$ that represents the higher value of the central tax rate that allows re-election in each locality. In our case, this value is equal to 11,92%. Imposing $r_c^*$ to the three localities, the revenue of the central government is equal to 25,032. As we found before, if it chooses the re-election in 2 localities out of 3 its tax revenue is equal to 44,99. Therefore the central government will choose the “2-out-of-3” solution.

If we change our initial hypothesis, we may show that the choice of the central government varies. Let us consider three re-election constraints: $\tau_1 = 0,5$, $\tau_2 = 0,295$ e $\tau_3 = 0,29$. We compute that the “2-out-of-3” solution leads to a tax revenue of 41,73. If we consider the unanimity solution the value of $r_c^*$ is 21,82% and our tax revenue is 41,78.

The choice between the “2-out-of-3” solution and the unanimity solution (3 out of 3) is based on the re-election constraints. Starting from our hypothesis we show that governments may choose not to maximise their citizens’ consensus, because, as in our first simulation, the 2 out of 3 solution may represent the best scenario (the highest rent) for the central government.
8. Central public good and local public good

Up to now we have dealt with a particular case: the two taxes (central and local) were perfect substitutes. This fact means that the line $\tau^*\tau^*$ in figure 1 could represent two different situations: the presence of a single public good that can be provided by one of the two level of government alternatively or the provision of two public goods, a local one and a central one, that are perfect substitutes (or, rather, are perceived as perfect substitutes by the citizens).

We now assume the presence of two different public goods and we suppose that there exists a level of substitution between these two goods, even if it is not perfect. In fact, we assume that the provision of a mix of public goods is preferable to the provision of a single good. Then citizens are willing to pay different levels of taxes for different proportions of the two public goods. In particular, we assume that citizens accept higher levels of taxation in the first case than in the second one.

Our median voter chooses the level of public goods through a benefit-cost analysis. He maximises the difference between benefits and costs.

In this case our analysis forecasts a locus of points that represent the different optimal combinations of local public goods and central ones.

We can represent the benefit of the local public good for the median voter using the level of taxes:

$$B(G_L) = f(\tau_L Y) \quad (33)$$

where $f' > 0$ and $f'' < 0$ because we suppose that an increase in the quantity of the public goods increases the welfare of the median voter and that the marginal benefit is decreasing for higher levels of the good. At the same time, the cost of the local public good is:

$$C(G_L) = g(\tau_L Y) \quad (34)$$

where, obviously $g' > 0$ and $g''$ can be equal or lower than 0 depending on the level of distortion.

The figure 3 represents equations (33) and (34) with $g'' = 0$. 
The utility of our median voter is as follows:

\[ U(G_L) = B(G_L) - C(G_L) \] (35)

We know that median voter will demand local public goods if \( B'(G_L) \geq C'(G_L) \) and he will maximize his utility when \( B'(G_L) = C'(G_L) \). We assume the presence of a single tax rate \( a \) where \( B'(G_L) = C'(G_L) \) in the interval \( 0 < \tau_L < c \), where \( c \) represents the tax rate that maximises the tax revenue for the local government. Furthermore, we assume that in \( 0 < \tau_L < a \) \( B'(G_L) > C'(G_L) \) and in \( a < \tau_L < c \) \( B'(G_L) < C'(G_L) \).

It is immediate to note that in a single-good market (the local one) the tax rate chosen by the median voter to re-elect the local government is \( a \).

The same kind of reasoning can be used to analyse the presence of a central public good as the unique public good. In this case we name our central tax rate \( b \).

If we have only one public good we can note that the tax rate that allows the re-election is \( \tau_L = a \) with \( \tau_C = 0 \) and \( \tau_C = b \) with \( \tau_L = 0 \), where \( B'(G_i) = C'(G_i) \) with \( i = L, C \).

We want to analyse the behaviour of our median voter when the two goods are provided simultaneously.

We assume that a solution where the two goods are provided (\( \tau_L \) and \( \tau_C \) both positive) may be preferred as compared to a solution where only one good is provided.

Equation (25) can be useful to represent what we said before:

3 The equations \( B(G_L) = k \sqrt{\tau_L Y} \) and \( C(G_L) = \tau_L Y \) assuming \( Y = (Y_0 - \tau_L Y_0) \) can represent an example to describe our situation.
\[
\frac{\tau_L^\sigma}{a^\sigma} + \frac{\tau_C^\sigma}{b^\sigma} = 1
\]  

(36)

Equation (36) says that \( \tau_C = b \) if \( \tau_L = 0 \) and \( \tau_L = a \) if \( \tau_C = 0 \). Moreover we suppose that \( \sigma \) is higher than 1, because our median voter prefers a solution where both goods are provided, then we have a concave constraint\(^4\).

We can represent equation (36) in figure 4:

\[\text{Figure 4}\]

\[\begin{array}{c}
\begin{array}{c}
\tau_C \\
\tau_L
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
b \\
a
\end{array}
\end{array}\]

Also in this case each level of government chooses to maximise its tax rate as shown in the sequential game described in the third section. As we assumed before, the central government maximizes its tax revenue leaving to the local one the tax revenue that allows the re-election.

We consider the new re-election constraint as modelled in equation (36) and an income-distortion function as:

\[
Y = Y_0 - (\tau_L + \tau_C)Y_0
\]  

(37)

where \( Y \) represents the taxable income and \( Y_0 \) the income without any distortion (with \( \tau_L + \tau_C = 0 \)). Our central government maximises its tax revenue as before. The local government earns that rent that leads it to behave yieldingly. Equation (38), starting from (6), (36) and (37), shows this condition:

\[\text{\footnotesize\textsuperscript{4} In the previous section we assumed that our re-election constraint was a particular version of (25) with } a = b = \tau^* \text{ and } \sigma = 1.\]
\[
\left(1 + \frac{1}{r}\right)\tau_L Y_0 - \left(\tau_L + \tau_C\right) Y_0 = \tau^{FS} Y_0 - \left(\tau^{FS} + \tau^{LS}\right) Y_0. \tag{38}
\]

From (35) we can express the value of \( \tau_C \) given \( \tau_L \):

\[
\tau_C = \left(\frac{b^\sigma}{a^\sigma} - \frac{b^\sigma}{a^\sigma} \tau_L^{\sigma}\right)^{\frac{1}{\sigma}} \tag{39}
\]

making a substitution in equation (38) that can be rewritten as follows:

\[
\left(1 + \frac{1}{r}\right)\tau_L Y_0 - \left(\tau_L + \frac{b}{a} \left(a^\sigma - \tau_L^{\sigma}\right)^{\frac{1}{\sigma}}\right) Y_0 = \tau^{FS} Y_0 - \left(\tau^{FS} + \tau^{LS}\right) Y_0. \tag{40}
\]

Equation (40) defines implicitly the value of \( \tau_L \) that maximises \( \tau_C \).

Competition between the two levels of government\(^5\) implies that the sequential game described in section three still represents the scenario, even if we consider a non-linear re-election constraint.

---

\(^5\) \( \left(1 + \frac{1}{r}\right)\left(\tau_L + \tau_C\right) Y_0 - \left(\tau_L + \tau_C\right) Y_0 > \left(\tau^{FS} + \tau^{LS}\right) Y^S \)
9. Conclusions

In this paper, we analyse the competition between two levels of government to maximise their rent. In particular, we analyse the competition when information is incomplete and citizens are only interested in the sum of the taxes they must pay to the two levels of government.

We consider four different scenarios. In the first one, preferences are the same for each locality, taxes are proportional and there is a linear constraint. Instead, the second scenario considers progressive income taxes. In the third, preferences are different in different localities, while in the fourth, we develop a non-linear constraint.

In the first scenario we provide a model where the two levels of government compete to maximise their rents subject to their re-election constraint. In particular, we show that a government maximise its rent for infinite periods if it leaves a rent to the other one high enough to convince it to behave yieldingly.

In the second scenario, after the assumption that the central government is the leader, we show that a progressive central income tax may lead to a lower difference between the central and the local rent.

The third scenario is useful to show that in a situation where localities have different preferences the central government may choose not to maximise its consensus and loose the election in a non-pivotal locality. Local governments of localities that accept a high level of taxes earn a high rent as compared to local governments with a lower constraint.

In the fourth scenario we introduce a higher level of information among citizens. They can distinguish between local public goods and central public goods and they may choose different combinations of these two goods. Nevertheless, governments compete to maximise their revenue as before even if they face a non-linear constraint.

In this paper we do not consider competition with perfect information. In that case the median voter has two different constraints for each government. In further research we will analyse this case. However, the hypotheses of this paper can be used to describe the real world, where people have perfect information about taxes and incomplete information about public goods and public services.
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