Choosing the electoral system: why not simply the best one?

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Abstract. The paper illustrates a simple empirical rule to choose the best electoral system for a Parliament.

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1. Introduction.

At first sight, the problem of finding the best electoral system for a Parliament cannot be solved, for three main reasons. First, some theorems—Arrow's and McKeveley's in primis—exclude the very possibility of finding out the optimal rule. Second, there are too many dimensions involved (see below): it may be too difficult to balance all of them. Finally, the adoption of complex methods appears at odds with the necessity of adopting a rule sufficiently easy to be understood and managed by voters. Improvements too technical or too cumbersome (or both) are unlikely to command the interest of policymakers.

The objections are not compelling. It is true that it is impossible to find out the optimal rule, but no theorem prohibits to find out an empirical criterion to establish whether a rule is better or worse than another. It is true that the electoral system affects a lot of dimensions, but there is a large consensus that some are more important, and it makes sense to consider only them. Finally, quite complex rules may be managed and understood by voters. The panoply of methods actually adopted here or there is very large, and the reduction of the choice set to them rules out the objection.

It follows that it is possible to find a solution for the problem of choosing the best electoral system, albeit on empirical grounds. The meaning of “solution” in this context will be stated in a moment. Before that, we must observe that there is no reason why a system should be always the best one. In a very divided society, where the political axis is very long, majoritarian systems may be very dangerous, as many people may feel too under-represented, possibly to the point that the democratic fabric of the society may be at risk (see f.i. Lijphart, 1999). On the opposite, in a very compact society there is no need to give away the efficiency of majoritarian Parliaments in exchange for the small increases in representation allowed for by proportional rule. From the point of view of public choice, this simple fact splits the problem in two. On one side, there is the problem of choosing the right electoral system in a given situation. On the other, there is the problem of the optimal rule for choosing the electoral system. The rule should take into account the specificity of given situations.

The second problem is obviously more important, as it includes the first. I suggest that a choice rule cannot be considered a useful solution for our problem unless three requirements are satisfied. First, for a system to be judged "good", and hence "better" or "best", it must be evaluated with reference to some criteria, to be provided by the rule. Second, the criteria must be such that the political system may actually employ them, through a sensible decision process. Third, the choice of the winning system and its implementation must be viable for political institutions. In other words, the solution to the problem "how to choose the best electoral systems for a Parliament" is made of three steps: to find an evaluation criterion, to choose the system that best fits with it, and to implement the chosen system. If one or more of these steps are too cumbersome, to the point that the political system cannot reasonably manage it, the solution is not viable.

In this paper I will suggest a solution that appears viable. It is valid if the relevant dimensions may be reduced to two. It should not be too difficult to elaborate an analogous rule for more dimensions, but there is a general agreement that the relevant ones are actually two. As the reader will see, the rule is simple and easy both to implement and to understand: all is requested is the statement of the relative weight of the two dimensions.

The paper is structured as follows. Section 2 describes the relevant dimensions and the way to assess the performance of an electoral system with reference to them. Section 3 explains how to compare electoral systems and how to choose the best one. Section 4 argues that the choice may best be made after the vote. Section 5 contains an illustration. Conclusions and caveats follow.
2. Two dimensions.

The choice of the electoral system affects a lot of facets of the political process. A quick survey of recent literature is sufficient to produce a fourteen, possibly sixteen item list: the representation of voters' wishes; the efficiency in governing; the degree of corruption (Myerson, 1993 and 2001; Persson et al., 2001); the dimension, type and timing of public spending (Persson and Tabellini, 1998 and 2001; Milesi-Ferretti et al., 2000); the overall welfare of a country (Mueller and Stratmann, 2000); the information and the participation of voters (Mudambi et al., 1995 and 1999); the responsiveness of the government's choice to the preferences of the voters (Shugart, 2001); the relative power of the lobbies (Myerson, 1995); the incentives for politicians (Myerson, 1995; Ryker, 1982); the possibility of strategic choices of voters and candidates (Levin and Nalebuff, 1995); the complexity of the voting system (id.); the protection of the minorities (id.; Rae, 1995; Sen, 1995); the risk of extreme choices (id.); the use of the vote as a "voice" device (Brennan and Hamlin, 1998; Sen, 1995). For the time being, I did not find references to two further dimensions largely quoted in the political debate, at least in Italy: economic growth and the reshuffling of the political aristocracy.

This list is quite formidable, but fortunately we can rest on the general agreement that two of them are of paramount relevance, that is the efficiency in representing electors' will and the effect on the efficiency of the resulting government. I'll label the first dimension representativeness, shortened with R, and the second one governability, G, from the Italian neologism "governabilita". There are good reasons to privilege R and G. To choose the representatives and to summon a government are the basic aims of the electoral process. I've suggested elsewhere (see Ortona, 2002) that minor dimensions may be better dealt with in other moments of the political process. In addition, it is sensible to think that albeit there is a trade-off between G and R, other dimensions are lexicographic with respect to them. If this is so, the results we'll obtain here will keep their validity irrespective of the dimensions judged relevant. In any case, what follows deals only with the two quoted dimensions. The extension of the results to the case of three or more is a possible and interesting task - deferred to further inquiry.

G and R may be evaluated through the assessment of numerical indicators, hopefully plausible but unavoidably arbitrary. I will label them g and r respectively. One should not be shy to suggest that electoral policies may be based on the values of indicators: this is what happens every day in economic policies, frequently oriented by changes of decimals in quite rough indicators like aggregate inflation or GNP growth rate. A detailed description of the indicators I will employ here is in appendix a. Briefly, g depends on the number of parties and of Mps supporting the government, and r depends on the difference of seats attributed to parties under the system under exam and under proportional representation, supposed to be the most representative system. The range of both is the interval 0-1.

After the voting, it is obviously possible to simulate the Parliament that will result under all systems allowed for by the information collected through the votes (for instance, to assess the performance of Condorcet voting or of Borda count we need more than the first preferences of the voters); and consequently to evaluate g and r for each of them. Results may be graphed, as in fig. 1.

There are three possibilities. First, a system may locate northeast of all the others, like the system labelled with ?. I define this system dominant, and it is obviously the best one. Unfortunately, this system is very likely not to exist; in all the simulations I have made so far (see Ortona, 1998, 2000 and 2002; Bissey and Ortona, 2003) it did not manifest itself – hence the symbol. Second, a system may locate southwest of at least another, like system 3 with reference to

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1 All previous references to be read as "for instance".
2 In what follows, I assume that majorities are always minimum winning coalitions of parties, and that coalitions must be made of parties adjacent on the left-right axis. These hypotheses may be substituted with more sophisticated ones in real-world applications of the rule, albeit they look plausible and are commonly (but not without discussion) accepted in the literature.
system 2 (and to system ?, if it exists). I define such systems \emph{dominated}, and they may safely be excluded: no need to consider system 3, if system 2, better on both dimensions, is available. Third, systems may be neither dominant nor dominated, like 1 and 2 in the graph (if ? does not exist), and like (usually) majoritarian voting and proportional representation in real world. I label these systems \emph{alternative}. Obviously, the rule we look for is useful only if it allows to choose among alternative systems.

\emph{Figure 1.} Plotting of five hypothetical electoral systems in the g-r space.

3. Choosing among alternative systems.

In order to compare the results of different electoral systems, we obviously need \emph{votes} for different systems: a majoritarian vote, a proportional vote, a list of voters' ordered preference for Condorcet voting, and so on. For the moment, suppose that we actually have these data; we'll come back to this in the next section. Given the votes, every system considered will produce a potential Parliaments, and each of them will have a pair of values of \(g\) and \(r\). If a system will result \emph{dominant}, it is the good one, but as we noticed this result is very unlikely, given the trade-off between the two dimensions. What can safely be done is to rule out \emph{dominated} systems, like system 3 in the previous figure. This operation will probably reduce considerably the number of potential candidates. Suppose that we remain with three, like in fig. 1 if system ? does not exist. Apparently, what we need to compare them is a social utility function (SUF) -admittedly a quite formidable requirement, to say little. Actually, we may be satisfied with something less.

Let us admit that the the SUF for electoral system is a Cobb-Douglas function in \(g\) and \(r\), \(U = Ag^a R^b\). We choose this form not only for its simplicity and versatility, but also for the meaning of \(a\) and \(b\), the partial elasticities of \(U\) with reference to \(g\) and \(r\) respectively. Now consider two systems, \(X, Y\). We may write that

\[ U_X > U_Y \iff Ag^a R^b > AG^a R^b \]

where we denote for simplicity the values of \(X\) with lower-case letters and those of \(Y\) with capitals. In addition, let \(p = a/b\), and hence \(a = bp\).

\[ [1] \quad U_X > U_Y \iff (g/G)^p > (R/r)^b \]

hence the condition may be written as
\[ p \ln(g/G) > \ln(R/r) \]

or

\[ p > \frac{\ln(R/r)}{\ln(g/G)} \]

provided that \( g \), or \( G \), or both are <1\(^3\).

The only \textit{a priori} information we need to assess the fulfilment of the condition is the value of \( p \), the ratio of the elasticities. I argue that this parameter may actually be provided by the political system. The ratio may be considered a proxy of the relative weight that the community assigns to an increase in the relative value of \( g \) and \( r \). If for instance a -say- 10\% increase in \( g \) is valued more than the same increase in \( r \), \( p>1 \), and viceversa. I suggest that the Supreme Court -or someone else at a very high level- may decide the value of \( p \). Alternatively, someone (but not the Parliament itself) may vote on its value. There are other possibilities. For instance, the community may "start" with \( a=b \), and change the ratio for the next election if the entity in charge thinks that representativeness or governability have been excessive. Another possibility is that the community orders the systems according to their supposed degree of proportionality, and chooses the first one (i.e. PPR) if the value of \( g \) is above a given threshold; otherwise it moves to the second one, and so on. Under this rule, the best system is the most proportional one, provided that the governability reaches a given value (or viceversa, obviously). I will not go deeper into this: what I claim is only -but it is not that little- that the problem of choosing the best electoral system may reasonably be reduced to that of choosing the relative importance of the two main dimensions\(^4\).

\[ [2] \] Allows for binary comparisons of (non-dominated) electoral systems, and hence to find out the Condorcet winner\(^5\). The winner is the best system, to be adopted.

Alternatively, we may trace indifference curves and pick the system lying on the higher one, as follows.

The expression for the generic indifference curve \( r^*=(U^*/g)^{1/b} \) is

\[ r = \left( \frac{U^*}{A} \right)^{1/b} g^{a/b} \quad \text{i.e.} \quad 3' \]

\[ r = \frac{W}{g^p} \]

\[ 3 \] This is the condition when \( g>0 \) and \( R>0 \). If \( g>0 \) and \( R>0 \), or \( G>0 \) and \( R>0 \), or \( G=0 \), or \( R=0 \), the choice is trivial. If \( G>0 \) and \( R=0 \), the sign of inequality \([2]\) becomes <.

\[ 4 \] Note that \( p \) may also be characterized in another, more suggestive way. \( p \) is the \textit{price} in terms of a relative decrease of \( r \) that the community accepts to pay for a given relative increase of \( g \) (and \( 1/p \) the opposite). If for instance \( p=2 \), it is worthwhile to accept a 20\% reduction of \( r \) to gain a 10\% increase in \( g \). The proof follows.

from \( U = Ag^{gb} \) and \( a = pb \) we get

\[ dU = dg(bpAg^{bp-1}r^b) + dr(bAg^{bp}r^{b-1}) \]

If \( U \) does not change

\[ 0 = dg(bpAg^{bp-1}r^b) + dr(bAg^{bp}r^{b-1}) \]

\[ dg(bpAg^{bp-1}r^b) = -dr(bAg^{bp}r^{b-1}) \]

\[ dr/r = -p(dg/g) \]

\[ 5 \] A Condorcet cycle may result only by chance, and may be ruled out simply by adding a further figure while rounding the results.
This provides an alternative way to compare couples of systems. For a given value of $g$, the value of $r$ increases with that of $W$, and the value of $W$ with that of $U^*$. Consequently, it is sufficient to solve equation [3'] for $W$, given $r,g$, and $p$, for each system considered. The system with the highest value of $W$ is the best one. If you are sufficiently able in computer plotting, the solution may appear graphically. An example is in fig. 2, where indifference curves for an hypothetical value of $p$ have been plotted across the (hypothetical) data of fig. 1. In absence of $\theta$, and ruling out system 3, dominated, the winner is system 2.

*Figure 2.* Choosing among four hypothetical electoral systems in the $g$-$r$ space.

![Diagram](image)

4. **Choosing the system after the vote.**

We left aside a main point, namely how to collect electoral results for different systems, in order to evaluate $g$ and $r$ for each. A possible solution is to run simulations with fictitious data corresponding (more or less) to the real features of the case. Another one is to draw on historical data to collect average data for the parameters. The limits of these approaches are obvious: the decision must be based on questionable data, hence the aim of the method -to choose a system as "objectively" as possible- is reached in a very limited way.

It seems much more advisable to choose the system *after the elections*. Electors may easily provide the information necessary to compute out the resulting Parliament under a lot of different systems, and hence $g$ and $r$. For instance, Ortona (2002) simulated the results of nine systems supposing that each voter provided a majoritarian vote, a proportional one and a complete ordering of preferences. To cast jointly the first two is common practice in Germany and in Italy, and the ordering of more relevant preferences (which would suffice) is requested wherever the single transferable vote (STV) is adopted. Note that this voting set allows for the simulation of additional systems, first of all the STV itself.

To choose the system after the voting may be very useful from another point of view too. According to Shugart (2001) and Shugart and Wattenberg (2000), mixed-members electoral systems are the best ones because, *inter alia*, they force the parties to try to maximize both the personal link of the candidate with the voters and the validity of nation-wide programs. The same result may be obtained if the parties do not know what will be the relative weight of the majoritarian and of the proportional components. More generally, if parties, and other concerned subjects, like lobbyists, do not know with certainty which system will ultimately be adopted, they will have less room to exploit the
pitfalls of each systems. Obviously, to choose the system *a posteriori* is possible only if the rule established by electoral norms provides undisputable results -as is the case here.

5. An illustrative experiment.

Just to give an example, in this section I will illustrate the results of some simulations, run at the Laboratory of experimental and simulative economics of the University of Piemonte Orientale. The Parliament is supposed to have 100 members. The data set is a representative, nation-wide survey of the complete preferences for (then) existing parties of Italian voters in 1997. It is useful to stress that the Italian electoral system was perfect proportionality (with an average district magnitude close to 20) until 1994, and has then been changed into a mixed-member system. Italian voters are consequently requested to vote for both a majoritarian and a proportional competition, in an environment characterised by a high number of parties. The simulation program allows presently to consider eleven systems. The method employed is that of equation [3'] above; hence the crucial figure is the value of W. Results are summarized in table 1.

**Table 1.** Summary of the simulation of 11 electoral systems.

<table>
<thead>
<tr>
<th>System</th>
<th>w values of p:</th>
<th>r</th>
<th>g</th>
<th>Seats of the governing coalition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional</td>
<td>0.36 0.13 0.60</td>
<td>1</td>
<td>0.36</td>
<td>58</td>
</tr>
<tr>
<td>Threshold prop. °</td>
<td>0.33 0.12 0.54</td>
<td>0.893</td>
<td>0.37</td>
<td>61</td>
</tr>
<tr>
<td>Prized prop. +</td>
<td>0.43 0.26 0.55</td>
<td>0.714</td>
<td>0.60</td>
<td>60</td>
</tr>
<tr>
<td>First-past-the-post</td>
<td>0.29 0.18 0.37</td>
<td>0.464</td>
<td>0.63</td>
<td>63</td>
</tr>
<tr>
<td>Two-round</td>
<td>0.29 0.18 0.37</td>
<td>0.464</td>
<td>0.63</td>
<td>63</td>
</tr>
<tr>
<td>Condorcet</td>
<td>0.29 0.18 0.37</td>
<td>0.464</td>
<td>0.63</td>
<td>63</td>
</tr>
<tr>
<td>Borda</td>
<td>0.29 0.18 0.37</td>
<td>0.464</td>
<td>0.63</td>
<td>63</td>
</tr>
<tr>
<td>Approval</td>
<td>0.30 0.11 0.48</td>
<td>0.786</td>
<td>0.38</td>
<td>64</td>
</tr>
<tr>
<td>Mixed-member 1 *</td>
<td>0.34 0.20 0.44</td>
<td>0.571</td>
<td>0.59</td>
<td>59</td>
</tr>
<tr>
<td>Mixed-member 2 #</td>
<td>0.32 0.12 0.54</td>
<td>0.893</td>
<td>0.36</td>
<td>58</td>
</tr>
<tr>
<td>VAP §</td>
<td>0.56 0.35 0.70</td>
<td>0.882</td>
<td>0.63</td>
<td>58</td>
</tr>
</tbody>
</table>

Cues:
° 5% threshold.
+ the majority prize is 15 seats.
* 25 seats elected under proportional system, 75 under first-past-the-post.
# 75 seats elected under proportional system, 25 under first-past-the-post.
§ VAP is a suggested new system. It is described in detail in Ortona, 2000 and 2002. a summary is in appendix b.

Remember that if p>1, governability is more appreciated than representativeness (see the text), and viceversa if p<1.

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6 A first version of the program has been created by Roberto Trinchero, of the University of Torino (see Trinchero, 1998). The version employed here is the second, updated as a part of his Laurea dissertation by Monella (see Monella, 2002) at the University of Piemonte Orientale. A third version, completely renewed, is under construction (see Bissey and Ortona, 2003).

7 Data collected by ISPO, Milano.

8 The high number of parties may be the product of the long history of proportional representation, and this makes Italy a peculiar case for study. However, there are clues that the number of parties tend actually to be high whenever plurality or majority do not enforce the law of Duverger. An instance are the British elections of 2001, where more than 70 parties tackled the first-past-the-post competition.
With \( p = 1 \), the winner is VAP; if we rule it out, the winner is prized proportionality. Note that in general proportional systems perform better then non-proportional ones. Remarkably, the same holds for \( p=2 \): assigning governability twice the value of representativeness is not enough to make non-proportional systems preferable, at least for Italian voters. Not unexpectedly, if \( p=0.5 \) the best system (but for VAP again) is proportionality. Mixed members perform quite well, thus confirming some recent suggestions (see Shugart, 2001 or Shugart and Wattenberg, 2000).

6. Conclusions.

The conclusions reduce to two major claims and to a major caveat. The first claim is that it is possible to choose "objectively" the best electoral system, on the basis of the values of given indicators, as commonly done in economic policy making. The second claim is that it is possible and advisable to choose the electoral system after the polls.

Unfortunately, the caveat is mighty. The whole procedure rests upon the assumptions that the ratio between the (incremental) utility of the two relevant dimensions may be read as the ratio of the parameters of an utility function, and that the arguments of the function define it conveniently. The first assumption is a little demanding, but the second much more so. The values of the parameters, and hence the actual position of the function, as well as its derivatives, depend crucially on the specific form of the indicators. There is nothing "natural" in the formulas for \( g \) and \( r \) we employed here; and other formulas would provide different figures.

However, there are ways to reduce the importance of the objection. One is to resort to the "decision maker approach", as suggested above. Another one is to provide the decisors with examples of typical results, transforming the choice of the ratio between the elasticities into an experts' judgement. It is also possible to better standardize the indexes. All these matters require deeper inquiry, so they are postponed to further research.

Finally, the reader is advised not to consider the simulations of section 5 as conclusive. They are too limited to be more than illustrative. Further papers will be more conclusive.
Appendix a. The indicators for R and G

Index of representativeness, r.

The formula is

\[ r_j = 1 - \frac{\sum_{i=1}^{n} |S_{j,i} - S_{pp,i}|}{\sum_{i=1}^{n} |S_{u,i} - S_{pp,i}|} \]

where \( j \) refers to the electoral system, \( n \) is the number of parties, \( S_{j,i} \) is the number of seats obtained by party \( i \) under system \( j \), \( S_{pp,i} \) is the number of seats obtained by party \( i \) under perfect proportionality rule, and \( S_{u,i} \) is the number of seats obtained by party \( i \) if all the seats go to the largest party.

The index reads as follows. First sum: I assume that R is perfect under perfect proportionality rule, PPR. Hence the loss of representativeness incurred by party \( i \) is the (absolute) difference between the seats it would get under PPR and those actually obtained. Summing this loss across all the parties we obtain the total loss of R. Second sum: In order to normalize (0 to 1) this value, I divide it by the maximum possible loss of R. This maximum is obtained when “winner takes all” in a very strict sense, that is when the relative majority party takes all the seats. I-the ratio of the sums: up to now we got a loss of representativeness index, normalized in the range 0-1. Subtracting it from 1 we transform it into a representativeness index. The index is quite similar to one suggested by Mudambi (1996); the main difference is that it is normalized. However, it is not normalized across constituencies.

Example. Suppose three parties, L, C and R, in a parliament of 100 seats. Under PPR they obtain 49, 31 and 20 seats respectively, under majority (M) 90, 10 and 0, and under some other system (S) 60, 25 and 15. The value of \( r_m \) is 1- 82/102 = 0.196, and the value of \( r_s \) is 1- 22/102 = 0.784. That of \( r_{ppr} \) is obviously 1: 1- 0/102.

Index of governability, g.

Governability depends on the number of parties of the governing coalition that may destroy the majority if they withdraw, \( m \); and on the share of seats of the majority, \( f \). \( m \) is more important, so I add lexicographically the \( f \)-component to the \( m \)-component. Hence the index is made of the sum of two terms, the first related to \( m \), \( g_m \), and the second related to \( f \), \( g_f \). The range of the second term is the difference between successive values of the first: the term in \( m \) defines a lower and an upper boundary, and the term in \( f \) specifies the value of the index between them.

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9 The discussion of \( r \) is taken from Ortona, 2000.
10 The value of \( S_{u,i} \) is the total number of seats for the largest party, and 0 for all the others. If several parties are the largest ones ex aequo, I take the one which is the largest under most systems in the case considered.
11 Actually, to assume that the maximum of R corresponds to PPR is quite limiting. The number of parties that run under PPR depends upon a lot of factors, mostly upon the cost of running. If this cost is high, even PPR may correspond to a low “real” representativeness. Consequently, the representativeness of a given system is assessed only against representativeness under PPR for a given set or rules. This limitation may be serious if running costs (and other obstacles to participation of parties to the electoral competition) are important. Another objection that may come to mind is less relevant. The representation of minor parties is affected strongly by the number of MPs to be elected in every constituency. However, it is perfectly possible to assess the result of a virtual large-constituency proportional election to be used as a benchmark for the evaluation of \( r \), even if real ones are small. In this case, PPR with real district will appear as an additional system. With an unique constituency of 100 seats, as in this paper, the index suggested by Lijphart (1999, ch. 7) excludes only parties with less than 0.75% of the votes.
The limits defined by $g_m$ are simply $1/m$ (upper boundary) and $1/(m+1)$ (lower boundary). For instance, if the government is supported by just one party, $g$ is comprised between 0.5 and 1; if it supported by two $g$ is comprised between 0.333 and 0.5, and so on. Note that the addition of new parties produces smaller and smaller decreases in $g$, as it should be\(^{12}\). The number of seats of the majority coalition specifies the value of $g$ in the given range. The figure to be added to the lower boundary, $g_f$, depends from the lead of the majority coalition, according to the following proportion:

$$g_f/[1/m - 1/(m+1)] = (f-t/2)/(t-t/2)$$

from which

$$g_f = [1/m - 1/(m+1)] (f-t/2)/(t-t/2)$$

where $t$ is the total number of seats in the Parliament.

For instance, if there are 100 seats and the governing majority is made up of one party, and enjoys the support of 59 MPs, the value of $g_f$ is 0.09 ($9/50*1/2$). This value must be added to 0.5, to give a value of $g$ equal to 0.59. In sum, the formula for $g$ is:

$$g = g_m + g_f = 1/(m+1) + [1/m - 1/(m+1)] (f-t/2)/(t-t/2)$$

where $m =$ number of crucial\(^{13}\) parties supporting the Government, $f =$ number of seats of the majority, $t =$ total number of seats. The maximum value of $g$ is 1, when a party has all the seats; the lowest is close to $2/t$, when all the parties have one seat, sufficiently close to 0 to justify the claim that the range of $g$ is the interval $(0,1]$ (actually, the lower limit of $g$ tends to 0 as $t$ tends to infinity).

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\(^{12}\) For instance, the difference of stability between a 7-party and a 8-party coalition is much smaller that that between a coalition of two parties and a single-party majority.

\(^{13}\) The number of crucial parties is the number of parties such that the Government is no more supported by a majority if one of them leaves the coalition. Under the hypothesis of MWC this number is equal to the number of parties that form the majority; but this is not true under the hypothesis of MWC of adjacent parties.
Appendix b. The VAP system

It runs through two stages. In the first, the Chamber is elected with a proportional system: perfect proportionality, threshold proportionality or single transferable vote. Here I assume perfect proportionality. The resulting majority forms the government, which gets its confidence vote as usual. From this moment on, the votes of the MPs belonging to the largest party(ies) and voting in accord with the government are given a weight $a > 1$. I label such party(ies) crucial. This is by and large equivalent to a majority premium: for instance, if $a = 1.5$, there are 100 seats, and the majority is made up of 60, the majority has up to 90 votes (see below), and the opposition only 40.

The rationale for this vote-weighting is to allow the government to maintain a lead even if small members of the supporting coalition defect. Small parties lose their blackmailing power. Only if large parties (or a sufficiently large portion of their MPs) defect the government loses the majority.

It follows that $a$ is computed according to the formula:

\[
(1) \quad a \sum_{i=1}^{m} X_i = \left( a \sum_{i=1}^{m} X_i + T - \sum_{i=1}^{m} X_i \right) / 2 + y \quad \text{i.e.} \quad a = \left( T - \sum_{i=1}^{m} X_i + 2y \right) / \sum_{i=1}^{m} X_i
\]

where $X$ is the number of seats of the $m$ largest parties supporting the government, and $T$ the total of seats. This way the government keeps a majority of $y$ if small members defect. If we want the government to be guaranteed if all members but the largest one defect, (1) reduces obviously to

\[
(2) \quad Xa = (Xa + T - X) / 2 + y \quad \text{i.e.} \quad a = (T - X + 2y) / X
\]

where $X$ is the number of seats of the largest party. Under this rule, a coalition would collapse if and only if a large member of it withdraws—which looks quite fair. Note that the number of crucial parties may also be decided on practical grounds. For instance, the electoral constitution may establish that it is worth “paying” a value of $a$ equal to 2.5 to obtain one crucial party, of 2 to obtain two, of 1.5 to obtain three and so on. If these thresholds are not reached, it means that correcting the (perfect or not) proportionality is too costly.

Example. Suppose a 100-seat parliament with 10, 10, 34, 17 and 29 seats for parties A to E. The Government is made by C and D. C is the only large party, so (2) applies, and the value of $a$ is 2. The majority enjoys 85 votes (34 times 2 + 17), and the opposition 49. If D withdraws, the Government keeps the majority, with 68 votes against 66. If it is C to withdraw, the former majority has only 17 votes against 83. Finally, the majority may collapse if some members of C defect: if D does not, at least 12 (or 13). With 12 defectors, the government has 61 votes (22 times 2 + 17) and the opposition 61 too.

In other words, it is not the real parliament to vote, but a virtual one, where the number of votes is given by those of the opposition, plus those of the small parties of the majority, plus those of the large parties of the majority times $a$, 2 in the example. If there are no defections, the number of virtual seats in the example is 134. Obviously, if a member of the majority votes for the opposition his 2 votes reduce to 1.

The index $g$ will be computed with reference to the virtual Chamber. In the example its value is $1/2 + (13/72)(1-1/2) = 0.59$. To compute $r$ we must create a perfectly proportional partition of seats in the virtual Chamber.

\[14 \text{ From Ortona, 2002.}\]
(see the formula); to do that we simply assign the virtual seats (134, in the example) to the parties according to their shares in the real Chamber.
References.
- RYKer, W. (1982), Liberalism against Populism, Freeman, S. Francisco, USA.

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